A simple (simplistic) method to include glaciated areas with a limited ice volume in the WaSiM and HBV models

Tómas Jóhannesson

5.12.2009

Background

Within the CES project, there is a need for simple runoff modelling of drainage basins with many glaciers without detailed mass balance and dynamic modelling of each glacier. Such modelling needs to take into account the limited ice volume stored in the glaciers, and preferably also the progressive lowering of the ice surface and the reduction in glaciated area that are associated with a reduction in ice volume. The current versions of the Swiss WaSiM and the Norwegian HBV models effectively assume an inexhaustible reservoir of ice with an unchanged altitude distribution, which may lead to an unrealistic contribution of melt water from glaciated areas in long integrations for a warming climate.

Glacier dynamics

This problem can be qualitatively analysed by considering the continuity equation for ice volume, which may be expressed as

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = b \quad \text{or} \quad \frac{\partial h}{\partial t} + \vec{\nabla} \cdot \vec{q} = b \;, \tag{1}$$

for a one-dimensional ice flow channel or an ice cap that flows in two horizontal dimensions, respectively. h is ice thickness, q or $\vec{q}$ is ice flux and b is mass balance. For (small) changes in glacier geometry with respect to a datum (often steady) state, perturbations in ice thickness, flux and mass balance will satisfy

$$\frac{\partial(\Delta h)}{\partial t} + \frac{\partial(\Delta q)}{\partial x} = \Delta b \quad \text{or} \quad \frac{\partial(\Delta h)}{\partial t} + \vec{\nabla} \cdot (\Delta\vec{q}) = \Delta b \;. \tag{2}$$

Changes in mass balance are the driving factor of glacier changes in climate change simulations. If the datum glacier is initially comparatively close to a steady state, changes in the ice flux gradient will be small to begin with because they are caused by accumulated changes in the glacier geometry over time. Mass balance changes will then to begin with to first approximation be balanced by corresponding local changes in ice thickness. According to the original full continuity equation, the ice flux will approximately balance the steady state mass balance by maintaining a "baseline" flow of ice from the accumulation area to the ablation area until significant changes in the glacier geometry have accumulated. The ice flux gradient term in the perturbation form of the continuity equation will then be close to zero and the terms $\partial\Delta h/\partial t$ and Δb on the left and right hand sides of the equation will approximately balance each other.

Since mass balance changes need time to bring about sufficient changes in ice geometry to alter the ice flow, one may initially assume that changes in the geometry of the glacier do not matter in runoff calculations so that the same values for the glaciated area and its altitude distribution may be used. This is the current implementation of glaciated areas in the WaSiM and HBV models. Negative mass balance perturbations in climate change runs are typically on the order of $-1\,\mathrm{m\,a^{-1}}$ so that over a period of a few or several decades, the ice surface may be lowered by several tens of metres. A lowering of that magnitude will start to affect the air temperature over the glacier and leads to an intensification of surface melting through the mass-balance–elevation feedback. Based on the preceding discussion, this effect might initially be taken into account by a local lowering of the ice surface, corresponding to the negative mass balance perturbation, without consideration of ice flow dynamics. This could be combined with a crude maximum on the total lowering based on an ice thickness estimate to take the limited ice volume into account. Local lowering of the glacier surface would, however, not take the retreat of the ice margin and the corresponding reduction in ice-covered area into account.

With some delay with respect to the ice volume reduction, the ice margin will start to retreat, which leads to a reduction in ice-covered area and runoff from the glacier. This effect counteracts the mass-balance–elevation feedback and brings about an approximately exponential decay of ice volume towards a new steady state of the glacier in the case of a "moderate" step change in climate.

The relative importance of the mass-balance–elevation feedback and the reduction in ice-covered area may be analysed with reference to the perturbation equation

$$\frac{d(\Delta V)}{dt} = B' + b_e\,\Delta A + G_e\,\Delta V = B' - \frac{\Delta V}{\tau_V}\ , \tag{3}$$

where the volume time-scale τ_V is given by

$$\tau_V = \frac{1}{(-b_e/H) - G_e}\ , \tag{4}$$

(Harrison and others, 2001). B' is the reference surface total mass balance (*i.e.* the specific mass balance integrated over the area of the whole glacier), defined as the mass balance corresponding to the reference or datum glacier geometry, b_e is the effective mass balance in the terminus region of the glacier (a weighted average over the area where changes in the location of the ice margin take place), G_e is the effective vertical mass balance gradient averaged over the entire area of the glacier (a weighted average over the vertical range over which changes in the ice surface elevation take place), and H is a scale for the ice thickness, defined as the gradient of volume changes to area changes, $H = dV/dA$, and assumed to be approximately constant over a substantial range in ice volume changes. The perturbations ΔV and ΔA are defined as differences with respect to a convenient choice of a reference glacier geometry which does not need to correspond to a steady state.

The runoff contribution due to ice volume changes is given by the term $d(\Delta V)/dt$ on the left hand side of Equation (3). The first term on the right hand side, B', corresponds to the climate forcing and is calculated without consideration to changes in altitude distribution or ice-margin position. The second term, $b_e \Delta A$, arises due to the advance or retreat of the ice margin and leads to a negative feedback, *i.e. less* runoff as glaciated area is reduced, while the third term, $G_e \Delta V$, represents the mass-balance–elevation feedback and leads to a positive feedback, *i.e. more* runoff as the ice surface is lowered and the rate of ablation is increased due to higher temperatures that drive the melting.

For maritime climates, a typical value for the vertical mass balance gradient, which represents the effect of the mass-balance–elevation feedback in the expression for τ_V, may be estimated as $G_e \sim 0.01\,\mathrm{m_{w.e.}\,a^{-1}\,m^{-1}}$, whereas the ratio of the magnitude of the mass balance at the terminus $-b_e$ to the ice thickness scale H, which represents the effect of the reduction in the ice-covered area, is on the order of $(-b_e/H) \sim 0.02\,\mathrm{m_{w.e.}\,a^{-1}\,m^{-1}}$. Thus, in terms of the effect on glacial runoff, the reduction in ice-covered area is typically more important than the mass-balance–elevation feedback when accumulated changes in glacier geometry start affecting the response of the glacier to a warming climate.

Size distribution of glaciers within a watershed

Measurements of ice thickness are typically not available for more than a few glaciers in drainage basins with many glaciers of different sizes as considered here. An estimate of the volume of ice stored in the glaciers therefore needs to be based on statistical considerations. Figure 1 shows the volume of the five largest ice caps in Iceland (Björnsson and others, 2008) together with a regression line through this data set and a regression line derived for a data set of more than a hundred valley glaciers (Bahr and others, 1997). The regression lines are of the form

$$v = c\,s^{\gamma}\,, \tag{5}$$

where v and s are glacier volume and area, respectively. The coefficient and exponent for the Icelandic ice caps are $c = 0.048$, $\gamma = 1.23$, when the area and volume are expressed in km^2 and km^3, respectively, and $c = 0.036$, $\gamma = 1.36$ for the valley glaciers. Figure 2 shows the same five Icelandic ice caps and the same regression lines together with volume–area data for eight ice caps and glaciers on Svalbard and in Scandinavia and the much larger ice sheets of Greenland and Antarctica.

Figures 1 and 2 show that reasonable approximations for the ice volume stored in glaciers and ice caps may be derived from data about the area distribution of ice bodies in the drainage basin. Since the exponents γ in the volume–area scalings $v = c\,s^{\gamma}$ for both ice caps and glaciers

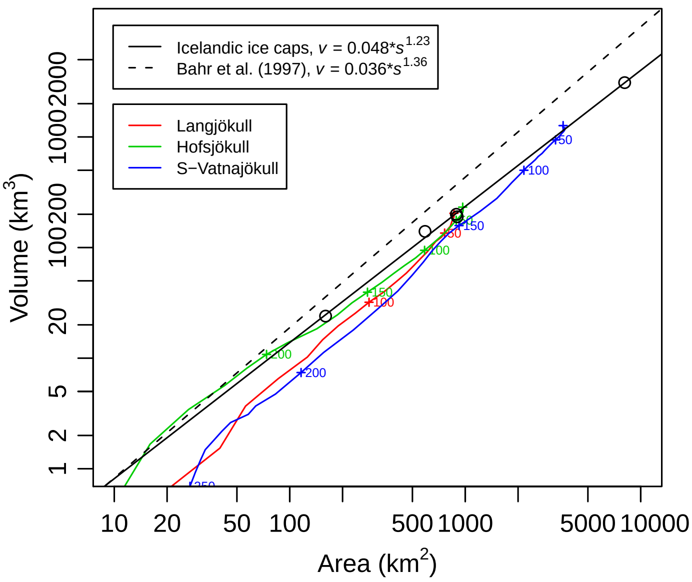

Figure 1: Volume and area of the five largest Icelandic ice caps (Vatnajökull, Langjökull, Hofsjökull, Mýrdalsjökull and Drangajökull, note that the symbols for Langjökull and Hofsjökull nearly coincide) (circles). The solid line shows a least-squares fit to the points and the dashed line the corresponding least-squares line derived by Bahr and others (1997) for 144 glaciers not including ice caps. The figure also shows numerical simulations of the volume and area for southern Vatnajökull, Langjökull and Hofsjökull (Jóhannesson and others, 2007) for a warming climate scenario developed as a part of the Climate and Energy (CE) project (Fenger, 2007). Symbols denote pairs of volume and area with 50 year intervals and labels show the time elapsed from the start of the simulation. Langjökull retreats more rapidly than Hofsjökull and Vatnajökull due to its "unfavourable" altitude distribution.

are comparatively close to 1, the scalings imply that the average ice thickness $h = v/s = c\,s^{\gamma-1}$ is a slowly varying function of the area. This means that, as long as the total glaciated area is well determined, some inaccuracy in the area distribution of individual glaciers can be accepted without causing a large error in the estimate of the total volume of ice stored in glaciers within the watershed.

A procedure for estimating the total ice volume in a watershed is outlined in Figure 3. A statistical estimate of the distribution of ice bodies of different areas within the watershed in question is first derived (Figure 3, left). The cumulative area distribution function $S(s)$ (shown on the x-axis) represents total area of all glaciers with area less than s (shown on the y-axis of the figure). Typically, the total glaciated area and an estimate of the area of a few of the

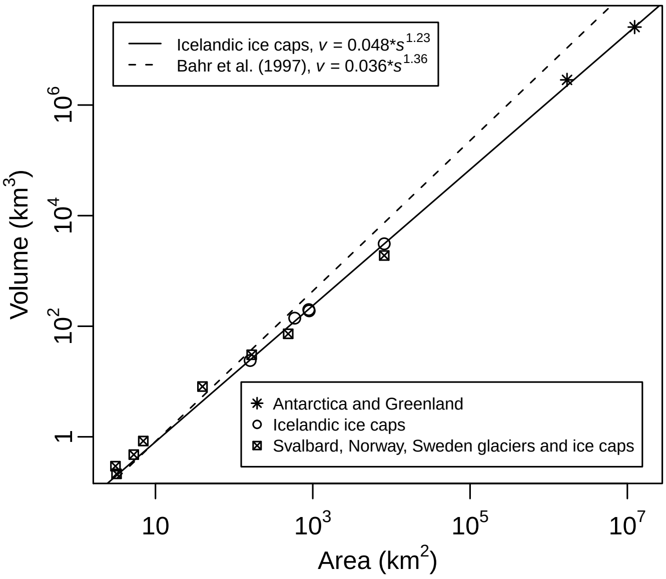

Figure 2: Volume and area of the five Icelandic ice caps (○) shown in Figure 1 together with the Greenland and Antarctica ice sheets ($*$) and the Austfonna, Jostedalsbreen, Trollbergdalsbreen, Engabreen, Storbreen, Langfjordjøkelen-east, Vestisen, Midtdalsbreen and Storglaciären ice caps and glaciers in Svalbard, Norway and Sweden(⊠). The solid and dashed lines are the same as in Figure 1.

largest ice bodies will be reasonably well known, giving a number of points on the curve near the top to the right. The shape of the area distribution curve near the lower left corner will be less well know and a sometimes a rough estimate based on available glacier inventories, aerial photographs and expert judgement must be used to complete the curve down to the origin of the figure. The volume distribution may then be computed by transforming the glacier area on the y-axis of the area distribution curve in Figure 3 to volume using Equation (5) resulting in

$$V(v_n) = \sum_{i=1}^{n} c s_i^{\gamma} \, , \qquad (6)$$

in case the cumulative area distribution function $V(v)$ is considered as a sequence of steps corresponding to n individual glaciers of different sizes (with volumes v_i and areas s_i for $i = 1$ to n), or using

$$V(v) = \int_0^v \frac{1}{\gamma} \frac{dS}{ds} \, d\xi \, , \qquad (7)$$

where the slope of the area distribution function, dS/ds, is considered as a function of ice vol-

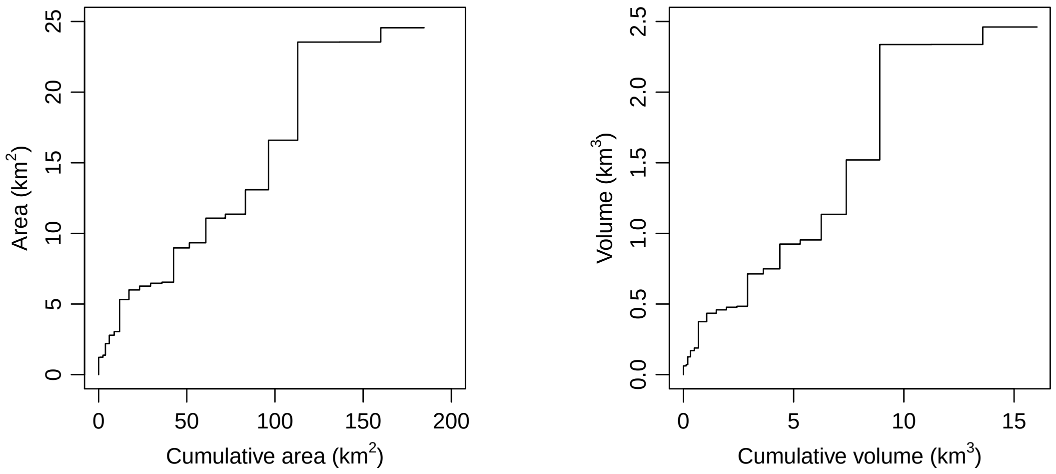

Figure 3: Hypothetical cumulative distributions of glacier area (left) and glacier volume (right) for a partly glaciated drainage basin with 20 ice bodies with randomly chosen areas in the range 1 to 25 km^2. The curves indicate the total area (volume) of glaciers with an area (volume) less than or equal to the value shown on the y-axis. The total glaciated area (total ice volume) is the maximum x-value reached by each curve. The area distribution (left) will in a practical case be determined from a glacier inventory for the watershed in question. The volume distribution (right) is derived by transforming the area distribution with Equation (6) or (7) (see text for explanation). Equation (6) with $c = 0.048$, $\gamma = 1.23$ was used in the example shown in this figure.

ume $v = c\,s^\gamma$ rather than area s, in case the distribution function is approximated by a continuous differentiable function. If the watershed contains both ice caps and valley glaciers, it is possible to use two area distribution curves, one for ice caps and another for valley glaciers. It is also possible to use a single area distribution function in this case and choose a weighted average of the volumes given by Equation (5) with the coefficients for ice caps and valley glaciers, respectively, with weights chosen based on the relative proportion of each type of ice bodies. It needs to be kept in mind that this procedure only provides a rough estimate of the total ice volume stored in the watershed so that details in the derivation of the volume distribution from the area distribution will in practice not alter the obtained total ice volume much in comparison to the overall uncertainty of the procedure.

Delayed response of the ice-covered area

The coloured curves in Figure 1, showing simulated volume and area for three retreating Icelandic ice caps with a shallow ice approximation ice flow model, demonstrate that the response of the ice-covered area to ice volume changes can be considerably delayed for rapid relative rates of ice volume change within the response time of the glacier in question. In this case, the ice volume is simulated to be reduced by over 50% in fifty years after the initial several decades with a slower response. All three curves show the same behaviour. They start out near the solid least-squares line in Figure 1 respresenting the relationship (5) betweeen ice volume and ice-covered area for Icelandic ice caps ($v = c\,s^{\gamma}$ with $c = 0.048$ and $\gamma = 1.23$). Over the first 50–100 years, the simulated decreasing ice volume becomes progressively lower than would correspond to the ice-covered area at the same point in time. The rapid glacier retreat thus results in an ice volume that is $\sim$20% less than predicted by the scaling relationship (5) during most of the simulation. The ice volume of Hofsjökull approaches the scaling relationship again near the end of the simulation after approximately 200 years when very little ($<$ 10%) ice remains. This is due to thick ice in a deep caldera near the centre of the Hofsjökull ice cap which takes a long time melt away.

The three numerical ice-cap simulations shown in Figure 1 indicate that an estimate of ice-covered area by inverting the volume–area scaling relationship (5) would result in a somewhat too small area which leads to too slow ice volume reduction after an initial period where this error may be expected to be small. This error is unavoidable when a time-dependent glacier model based on volume–area scaling is used but it is most likely within the relative magnitude of other errors in a basin-wide analysis as described here.

Model description

The model proposed here is intended to capture the most important characteristics of the cumulative response of many glaciers of different sizes within a single watershed to mass balance changes without the need for detailed dynamic modelling of each glacier. The focus is on glacier retreat, temporary advance is only crudely modelled. The main features of the model are

- Glaciers within the modelled watershed are divided into one or more groups that are treated together more or less as one ice body each in the simplified glaciological modelling.
- The total ice volume is separately estimated for each group of glaciers so that the runoff from the corresponding glaciated part of the watershed is limited by the initial ice volume estimate for the group.
- The average ice thickness and the glaciated area of each group are assumed to be reduced according to the scaling relationship (5) in a manner that is thus roughly consistent with glacier dynamics, thereby reflecting the delayed response of glaciers to climate changes.
- The reduction in glaciated area is assumed to take place mainly at the lowest altitudes due to the retreat of the ice margin at the terminus or at multiple termini of outlet glaciers. The distribution of the reduction in ice-covered area between the lowest and higher ice-covered elevations is determined in a manner that is consistent with an estimate of the response time of the glaciers (see further details below).

- The lowering in the altitude of the ice surface is explicitly accounted for, thereby taking into account the mass balance–elevation feedback, potentially both the feedback due to ablation–elevation feedback and precipitation–elevation feedback depending on the formulation of the underlying hydrological/meteorological model.

The main simplifying assumptions of the model are

- The glaciated part of the watershed is for simplicity assumed to be covered by ice with uniform thickness, the total volume of which is estimated with the procedure outlined in the previous section.
- The mean ice thickness and glaciated area are assumed to be reduced according to relationships derived from the volume–area scalings described above as developed in more detail below.
- It is assumed that the effective delay in the response of all the glaciers in each group of glaciers to climate changes can be represented by a single time constant or response time for the whole group.
- During a temporary advance, the model first tries to advance the ice into the area which has been exposed by the retreat of the ice margin earlier in the simulation. If the advance is too great for this to be sufficient, the glaciated area is kept constant and only the average ice thickness is increased. This is a reasonable approximation for a cold spell that lasts only for a year or up to several years but becomes unsuitable for a long-lasting deterioration of the climate.

It is assumed that the watershed is divided into elevation bands or grid cells at different elevation levels, identified with an index i, preferably with a separate specification of the area and mean elevation for the ice-free and glaciated parts of each elevation band or grid cell. The areas are denoted by a_i and g_i, and the elevations by z_i and y_i, for the ice-free and the ice-covered areas, respectively. At the end of each hydrological year, the hydrological model will provide a simulated value for the total mass balance of each glacier group within the watershed, ΔV_a (the subscript "a" denotes change over the hydrological year under consideration), based on the simulated mass balance summed over all the corresponding glaciated elevation bands or grid cells. The total glaciated area, S_1, and ice volume, V_1, at the beginning of the year are assumed to be known, providing an estimate of the average ice thickness, $h_1 = V_1/S_1$. From the scaling relationship (5), the total change in glaciated area may be approximated as

$$\Delta S_a = \frac{1}{\gamma}\frac{\Delta V_a}{V_1} S_1 \ , \tag{8}$$

in case $\Delta V_a < 0$ or if $\Delta V_a > 0$ and there is sufficient previously ice-covered space to readvance the ice margin (and $\Delta S_a = 0$ in case $\Delta V_a >= 0$ and there is insufficient space to advance the ice margin). The change in average ice thickness is

$$\Delta h_a = \frac{V_1 + \Delta V_a}{S_1 + \Delta S_a} - h_1 \ , \tag{9}$$

in case $\Delta V_a < 0$, and $\Delta h_a = \Delta V_a / S_a$ for $\Delta V_a >= 0$ in case of an advance when there is insufficient space to advance the ice margin. The mean altitude of the ice-covered part of each elevation band or grid cell at the end of the hydrological year is then given as $y_i + \Delta h_a$. The increased ice thickness in case $\Delta V_a >= 0$ and there is insufficient space to advance the ice margin is not consistent with the volume–area scaling and needs to be melted away separately when the mass balance becomes negative again.

The only part of the model that remains to be specified is the detailed description of the reduction in the individual glaciated areas g_i in each elevation band or grid cell that together should sum up to the total reduction in glaciated area ΔS_a for the glacier group in question. Lacking an explicit representation of each individual glacier, this can only be done in a heuristic manner. Clearly the glaciated area should preferably be reduced at the lowest altitudes, but it should also be reduced to some extent at other altitudes since parts of the retreating ice margin will be located a higher altitudes. In order to capture both these aspects, the reduction of ice-covered area will be described as two components. A reduction in the glaciated area at the lowest altitudes is specified as

$$\Delta S_{a_1} = k \, \Delta S_a \ , \tag{10}$$

where k is an adjustable parameter, $0 < k < 1$. This reduction is carried out by eliminating the ice-cover sequentially, starting with the lowest elevation band or grid cell of the group and continuing upward until the total eliminated ice-covered area is equal to ΔS_1. Some ice may remain in the uppermost considered band or cell so that the eliminated area is exactly equal to ΔS_{a_1}.

A relative reduction in the glaciated area for all the remaining elevation bands with some ice cover is calculated as

$$\Delta S_{a_2} = (1-k) \, \Delta S_a = \sum_{i=1}^{n} \Delta g_i \ . \tag{11}$$

The reduction in the glaciated area Δg_i within elevation band i is assumed to be relative to the corresponding ice-covered area g_i and linearly proportional to the height difference between the band and the maximum altitude of the glaciers $y_{\max}$

$$\Delta g_i = r_i \, g_i \ , \qquad r_i = \frac{\Delta S_{a_2} \, (y_{\max} - y_i)}{\sum_{j=1}^{n} (y_{\max} - y_j) \, g_j} \ . \tag{12}$$

The second step is carried out after the lowest glaciated elevation bands have been altered by the reduction in the glaciated area at the lowest altitudes.

This procedure to distribute the reduction in glaciated area with altitude is clearly quite crude and the adjustable parameter k can only be determined in an *ad hoc* manner. An error in the distribution of the retreat of the glaciers with altitude will result in an error in the term $b_e \, \Delta A$ in Equation (3). If, for example, too much retreat takes place at the very lowest elevations, where the mass balance is most negative, the effect of the area reduction on the mass balance will be overestimated. This error will accumulate with increasing glacier retreat and lead to a wrong modelling of the feedback between ice-volume reduction and ice-covered area reduction. The value of k in Equation (10) can be chosen based on glacier dynamics to counteract this error as integrated over the whole glaciated area by using an (independent) estimate for the effective response time τ_V of the entire ice mass of the glaciers. Equation (3) may be written as an expression for τ_V in terms of the ratio between the total accumulated volume change since the start of the simulation, ΔV, to the difference between the annual volume change and the reference surface mass balance

$$\tau_V = \frac{-\Delta V}{(d\Delta V/dt) - B'} \ , \tag{13}$$

where rate of change of the ice volume per year is $(d\Delta V/dt) = \Delta V_a$.

If a rough estimate of τ_V for the glaciers in question can be made, based on mass balance and ice thickness estimates for the glaciers in the group, Equation (13) can be used to make corrections to k. If the response time according to Equation (13) is too low, Equation (4) indicates that b_e is too negative and the glaciated area has thus been reduced too much at the lowest altitudes. Then the value of k needs to be reduced in order to correct the error. This argumentation needs to be applied in an iterative fashion at the end of each hydrological year so that response time estimates τ_V from Equation (13) are consistent with an *a priori* physical estimate of the response time of the glaciers. This procedure is not trivial to apply in practice. At the first time steps, the numerator and denominator of Equation (13) are both small numbers and the resulting estimate of τ_V is not useful. It is only after some changes in ΔV and ΔA have been simulated that the equation can be used to judge the values of k that have been chosen up to that point in time. Some iterations with initial guesses of k will need to be made in most practical cases. Estimates of τ_V from Equation (13) after substantial changes in ice volume and ice covered area ΔV and ΔA have been simulated may, furthermore, not be used to adjust k because the response time concept is based on a linearisation of the glacier dynamics and is not valid for changes that are relatively large compared to the initial size of the glacier. Here, it is suggested that the adjustment procedure for k described above is used until the ice volume has been reduced by on the order of one half and that a constant value of k is applied until the ice volume is completely exhausted.

The areas of the ice-free parts of the altitude bands or grid cells, a_i, and the mean elevations for the ice-free and glaciated parts of the bands or cells need to be updated based on the proposed values of Δh_a and g_i at the end of the hydrological year. This can be done with the following equations

$$a_i' = a_i - \Delta g_i \ , \quad g_i' = g_i + \Delta g_i \ , \tag{14}$$

and

$$z_i' = \frac{a_i z_i - \Delta g_i (y_i - h_1)}{a_i - \Delta g_i} \ , \quad y_i' = y_i + \Delta h_a \ , \quad \text{if} \quad \Delta g_i < 0 \tag{15}$$

or

$$z_i' = z_i \ , \quad y_i' = \frac{g_i (y_i + \Delta h_a) + \Delta g_i (z_i + h_1 + \Delta h_a)}{g_i + \Delta g_i} \ , \quad \text{if} \quad \Delta g_i > 0 \ , \tag{16}$$

where the primed quantities denote the updated end-of-hydrological-year values of the corresponding quantities at the beginning of the hydrological year. The average elevation of the whole elevation band is, furthermore, given as

$$\frac{a_i' z_i' + g_i' y_i'}{a_i' + g_i'} = \frac{a_i z_i + g_i y_i + (g_i + \Delta g_i)\Delta h_a + \Delta g_i h_1}{a_i + g_i} \ . \tag{17}$$

Discussion

The proposed model has two obvious flaws. First, uniform ice thickness is specified in the whole glaciated area for each group of glaciers and, second, the lumped retreat of the ice margin is a crude description of a much more complex reality. The first flaw may not be as serious as it might seem at a first glance. Glacier mass balance is often described with a constant or nearly constant mass-balance–elevation feedback. In that case, the detailed distribution of ice thickness within the glaciated area does not matter for the total mass balance integrated over the whole glacier. Furthermore, the initial altitude distribution of the ice-covered area will be reasonably accurate, as it may be assumed to be based on a map or an estimation of the initial glacier geometry. Inaccuracy due to the assumed uniform ice thickness will accumulate over time as the glacier geometry changes but will be rather small initially while changes in the glacier geometry are still small. When relatively large changes in the glacier geometry have taken place, the model will be crude anyway due to various other simplification and the most important model property is the proper conservation of ice volume, which the model should handle adequately. The simplistic handling of glacier geometry changes may, however, have a more significant effect on seasonal aspects of the glacial runoff. The model is likely to underestimate ice surface lowering at comparatively low altitudes and overestimate surface lowering at high altitudes. This will have an effect on seasonal melting and cause some flaws in estimates of changes in the seasonal distribution of glacial melt after significant changes in glacier geometry have occurred.

The simplistic specification of the reduction in glaciated area is likely to be more problematic than the uniform ice thickness. In reality, each individual glacier will retreat according to its own dynamics. The retreat will be most rapid at low elevations near the termini, but retreat will also take place at other locations along the ice margin. Small glaciers with a narrow elevation range will lose area at intermediate elevations within the elevation range of a large group of glaciers that contains much larger glaciers, but large and long glaciers will tend to lose area at much lower elevations. Small glaciers may also disappear more rapidly than large glaciers for rapid warming, but for small to moderate warming this difference between glaciers of different sizes will not be as great. This variability cannot be handled correctly without detailed modelling of each individual glacier, which the proposed model is intended to avoid. The strategy to distribute the reduction in ice-covered area with elevation so that a predefined value for the response time of the glaciers is maintained, as described in the previous section, should however lead to a roughly realistic description of the most important feedbacks.

The strategy to use a single physically-based estimate for the response time for each glacier group may seem to refute the original purpose of the model, which is to represent the variation of many glaciers of different sizes. A single value of the response time would seem to apply to only one of the glaciers and the others would not be appropriately represented by the model.

Equation (4) for the volume response time may be rewritten as

$$\tau_V = \frac{1}{G_e} \frac{H}{(\Delta Z_{T-ELA} - H)} \ , \tag{18}$$

where ΔZ_{T-ELA} is the difference in elevation between the equilibrium line altitude and an effective or weighted average altitude where retreat of the terminus takes place. Large glaciers tend to extend to lower elevations with respect to the ELA, that is have a larger ΔZ_{T-ELA}, which leads to a shorter τ_V according to this equation, but they also tend to be thicker, which leads to a longer τ_V (for similar values of G_e). The variation of τ_V within a group of glaciers that vary in size, may therefore be smaller than might be expected at a first glance. Large glaciers also have a tendency to be located on terrain with a more gentle bedrock slope than smaller glaciers which works in the same direction. This will in many practical cases limit the variation of the response time within a group of glaciers that vary in size over an order of magnitude to a much smaller relative range.

The assumption that area changes corresponding to calculated changes in ice volume can be estimated with the scaling relationship (5) is not necessarily true although the relationship provides a good fit to the volume–area distribution for a data set of glaciers (see Figures 1 and 2). Firstly, there is a delay between area changes and ice volume changes for rapid glacier variations as discussed above. Secondly, it is possible that a data set of glaciers would satisfy a scaling law at a particular point in time because of a relationship between glaciers of different sizes and landscape features of corresponding sizes, such as mountains and valleys, where glaciers tend to form. Since a glacier does not change location when it advances or retreats, such a relationship would not be expected to hold for time-dependent development in ice-covered area and ice volume for the individual glaciers. However, the mean ice thickness of an individual glacier or a set of glaciers may from glacier dynamics be expected to be related to areal extent to the power 1.5 for a valley glacier (one-dimensional flow system) and to the power 1.25 for an ice cap (cylindrically symmetrical flow system) on a comparatively flat terrain. For other terrain slopes similar relationships with somewhat lower exponents are expected to hold. Scaling relationships similar to Equation (5), with similar exponents as derived and quoted above, may thus be expected to provide an approximation for ice volume for time-dependent variation of individual glaciers as well as for the volume–area distribution of a group of glaciers of different sizes.

Acknowledgements

This study was carried out as a part of the CES project (“Climate, and Energy Systems”) financed by Nordic Energy Research (fund under the Nordic Council of Ministers) and the National Power Company of Iceland. Data about area and volume of glaciers and ice caps in Norway was provided by the Norwegian Water Resources and Energy Directorate (NVE).

References

Bahr, D. B., M. F. Meier and S. D. Peckham. 1997. The physical basis of glacier volume–area scaling. *J. Geophys. Res.*, **102**(B9), 20,355–20,362.

Björnsson, H., and F. Pálsson. 2008. Icelandic glaciers. *Jökull*, 58, 365–386.

Fenger, J. (Ed.). 2007. *Impacts of Climate Change on Renewable Energy Sources: Their role in the Nordic energy system*. Copenhagen, Nordic Council of Ministers, Nord 2007:003.

Harrison, W. D., D. H. Elsberg, K. A. Achelmaeyer and R. M Krimmel. 2001. On the characterization of glacier response by a single time-scale. *J. Glaciol.*, **47**(159), 659–664.

Jóhannesson, T., G. Aðalgeirsdóttir, H. Björnsson, P. Crochet, E. B. Elíasson, S. Guðmundsson, J. F. Jónsdóttir, H. Ólafsson, F. Pálsson, Ó. Rögnvaldsson, O. Sigurðsson, Á. Snorrason, Ó. G. Blöndal Sveinsson and Th. Thorsteinsson. 2007. *Effect of climate change on hydrology and hydro-resources in Iceland.* Reykjavík, National Energy Authority, Rep. OS-2007/01.

NAME

gls − lumped time-dependent modelling of ice caps and glaciers in one or more watersheds

SYNOPSIS

gls [options] parameter-file glacier-group-file0 glacier-group-file1
glacier-group-file2 elevation-band/cell-file0
elevation-band/cell-file1 elevation-band/cell-file2

DESCRIPTION

gls simulates changes in ice volume and ice-covered area from specific mass balance computed by a hydrological model such as WaSiM or HBV.

OPTIONS

-f Fixed *k* without updating with response time. The coefficient *k* specifies for each glacier group how area changes are divided between the lowest elevation cell and higher cells. By default the model adjusts the coefficient so that an estimate of the response time agrees with an estimate specified by the user in the initial glacier group file *glacier-group-file0*.

-r *rrv* Relative remaining volume when to stop updating the coefficient *k*. When the volume of a glacier group has been reduced below the value specified by this option (default 0.8) the coefficient *k* is kept constant and not updated using the given response time estimate.

-x *rlx* Relative steps in an iterative updating of *k* (default 0.5, see code for further explanations). The model tries to adjust *k* so that an estimate of the volume response time of each glacier group is close to the response time given in the input file *glacier-group-file0*. If the mass balance at the lowest elevations is not consistent with the specified response time the model uses the value of *k* from the last iteration or the value specified in the input file *glacier-group-file0*.

-m *mnk* Minimum value of *k* in the iterative updating (default 0.25)

-z *dhz* Delta *z* for calculation of highest altitude (default 50 m). This value is added to the altitude of the highest cell in calculations of the distribution of relative area reduction with altitude.

-d Print debugging information on stdout.

[-h|-H|-?] Print help text on stderr and exit.

-v Print program version on stderr and exit.

COMMAND LINE PARAMETERS

The input to *gls* is read from files with names that are given as command line parameters.

parameter-file

Various computational parameters. Each line specifies one parameter and consists of the value of the parameter as a floating point number followed by the three letter name of the parameter. The parameter value and the parameter name are separated by a space or a tab.

glacier-group-file0

Input file. Definition of one or more glacier groups in a reference state, for example at the start of the simulation period, one line for each group. The file has seven columns.
1: unique id-number of the group (integer).
2: name of the group (character without spaces).
3: type (character, "ic" for ice caps, "gl" for valley glacier).
4: total ice-covered area (floating point number, m^2).
5: total ice volume (floating point number, m^3).
6: characteristic response time (floating point number, years).
7: coefficient giving the relative area reduction that is to take place at the lowest elvation-band/grid-cell (floating point number, between 0.0 and 1.0).
In case each individual glacier is represented by a separate line in the file it is possible to let the model calculate the initial ice-volume with the volume-area scaling. Then the volume needs to be specified as "NA". This only makes sense if each area corresponds to an individial glacier rather than a group of glaciers that are lumped together.

Example:
1 hofsjokull ic 900e6 200e9 100 0.9

glacier-group-file1
Input file with the same column structure as *glacier-group-file0*. Total ice-covered area and total ice volume for each glacier group at the beginning of the hydrological year. The response time need not be given (the column should then contain "NA") and is ignored in case it is specified.

glacier-group-file2
Output file with the same column structure as *glacier-group-file0*. Total ice-covered area and total ice volume for each glacier group at the end of the hydrological year. The mass balance is not given.

elevation-band/cell-file0
Input file. Definition of elevation-bands or cells within glacier groups in a reference state, for example at the start of the simulation period, one line for each band or cell. The file has seven columns.
1: unique id-number of the band of cell (integer).
2: id-number of the glacier group containing the band or cell (integer).
3: sequential number indicating the relative order when bands or cells became ice-free.
4: total area (floating point number, m^2).
5: ice-covered area (floating point number, m^2).
6: mean altitude of the elevation-band or grid cell (m a.s.l.).
7: specific mass balace (floating point number, m water equivalent over the mass balance year).
The total ice-covered area of all elevation-bands or grid cells in a glacier group must be consistent with the area given in the glacier-group file. The mass balance in the file should be calculated for the reference geometry but with the climate parameters corresponding to the hydrological year that is being considered (same climate parameters as used to calculate the mass balance in *elevation-band/cell-file1*). Mass balance may be specified as "NA" for ice-free elements but the mass balance should be given for elements that the glacier may advance into in the next time step.
Example:
6 1 0 200e6 200e6 1250 -0.7

elevation-band/cell-file1
Input file with the same column structure as *elevation-band/cell-file0*. Area, ice-covered area, elevation and mass balance of elevation-bands or grid cells at the beginning of the hydrological year.

elevation-band/cell-file2
Output file with the same column structure as *elevation-band/cell-file0*. Ice-covered area and elevation of elevation-bands or grid cells at the end of the hydrological year.

PARAMETERS

The parameters that need to be specified in the parameter file are described below. An example value for each parameter with units where applicable is given after a short description of the parameter.

ggl Scaling exponent (γ) for ice volume/ice-covered area scaling for valley glaciers ($V = cA^\gamma$). Example: 1.36 (unitless).

cgl Multiplicative coefficient (c) for ice volume/ice-covered area scaling for valley glaciers. Example: 0.249 (assuming ice volume and ice covered area given in m^3 and m^2, repectively).

gic Scaling exponent (γ) for ice caps. Example: 1.23 (unitless).

cic Multiplicative coefficient (c) in the scaling relationship for ice caps. Example: 2.001 (volume and area given in m^3 and m^2, repectively).

idn Ice density. Example: 900 (kg m^{-3}).

INPUT

The input to *gls* is read from the files *glacier-group-file0*, *glacier-group-file1*, *elevation-band/cell-file0* and *elevation-band/cell-file1*.

OUTPUT

The output from *gls* is written to the output files *glacier-group-file2* and *elevation-band/cell-file2*.

INSTALLATION AND PORTABILITY

gls is written in *Fortran* for *UNIX* computers.

VERSION

Version 1.1 was issued in January 2010.

NOTES AND WARNINGS

The model is intended to work from very limited information about the geometry and altitude distribution of the glacier(s). For a retreating glacier, the model keeps track of the order in which cells become ice-free and fills same cells in the reverse order if the glacier later advances. The information about this ordering of the cells is stored in the sequential number in the third column of the elevation-band/cell files. The model is not able to advance the ice margin beyond cells that have been assigned a sequential number specifying the order in which they should be filled. When the last cell with a sequential number greater than zero has been filled, the glacier margin is not advanced farther and all further increase in ice volume is used to increase the ice thickness. This is unrealistic and the model issues a warning on stderr to notify the user when this situation arises. Users can specify the sequential number for cells close to the ice-margin in case they want to make it possible for the model to advance the ice-margin beyond the cells that initially contain some ice.

...

AUTHOR AND ACKNOWLEDGEMENTS

The model was programmed by Tómas Jóhannesson of the Icelandic Meteorological Office in the fall of 2009 as a part of the project CES ("Climate, and Energy Systems") financed by Nordic Energy Research (fund under the Nordic Council of Ministers) and the National Power Company of Iceland.

REFERENCES

Tómas Jóhannesson. 2009. A simple (simplistic) method to include glaciated areas with a limited ice volume in the WaSiM and HBV models. Reykjavík, Icelandic Meteorological Office, Memo ÚR-TóJ-2009-01.

AVAILABILITY

The *gls* software can be obtained from

Tómas Jóhannesson
Veðurstofa Íslands — Icelandic Meteorological Office
Bústaðavegi 9
IS-150 Reykjavík
Iceland
Phone: +354-1-5226000, Fax: +354-1-5226001, E-mail: tj@vedur.is